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Quark-hadron phase transition and strangeness conservation constraints

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Abstract. The implications of the strangeness conservation in a hadronic resonance gas (HRG) on the expected phase transition to the quark gluon plasma (QGP) are investigated. It is assumed that under favourable conditions a first order hadron-quark matter phase transition may occur in the hot hadronic matter such as those produced in the ultra-relativistic heavy-ion collisions at CERN and BNL. It is however shown that the criteria of strict strangeness conservation in the HRG may not permit the occurrence of a strict first order equilibrium quark-hadron phase transition unlike a previous study. This emerges as a consequence of the application of a realistic equation of state (EOS) for the HRG and QGP phases, which account for the finite-size effect arising from the short range hard-core hadronic repulsion in the HRG phase and the perturbative QCD interactions in the QGP phase. For a first order hadron-quark matter phase transition to occur one will therefore require large fluctuations in the critical thermal parameters, which might arise due to superheating, supercooling or other nonequlibrium effects. We also discuss a scenario proposed earlier, leading to a possible strangeness separation process during hadronization.

1 Introduction

The present day ultra-relativistic heavy-ion collision experiments at BNL and CERN offer us an opportunity to investigate the expected deconfinement phase transition from the hadronic matter to another phase of free or only weakly (perturbatively) interacting deconfined quarks and gluons called quark gluon plasma (QGP). Several phenomenological calculations indicate that this may happen at temperatures 150–200 MeV which probably can be achieved in the present day experiments. Lattice QCD calculations for baryon free matter indicate that a deconfining phase transition to an ideal non-interacting free QGP can take place at temperatures > 250 MeV [1,2]. The production of strange (antistrange) hyperons have been of great interest because it has been shown by theoretical models, during the last decade, that it may provide an unambiguous signature of the deconfining hadron-quark phase transition [3–6]. The formation of QGP is expected to enhance the abundance of strangeness relative to that obtained from the hadronic interactions at the similar energies. The present NA35 and WA85 CERN experimental data have indeed shown an enhancement of about 3–4 times in the strangeness production in the nucleus-nucleus collisions when compared to the corresponding pp collisions. However, these experimental observations have been explained by assuming the existence of an equilibrated hadronic resonance gas (HRG) at the freeze-out [7–13]. In addition to this one may also require to assume that the strange sector of the HRG may be only in a partial chemical equilibrium [7–13]. These HRG models describe the hadronic matter in terms of a gas of hadronic resonances with different masses where individual mass states are populated according to the equilibrium Fermi-Dirac or Bose-Einstein distributions. Furthermore, since we are dealing with a system of strongly interacting particles, quantities such as baryon number and strangeness are conserved. Since the total number of particles in the system is not fixed due to creation, annihilation and other reaction processes, thus this is achieved within the framework of the grand canonical ensemble by introducing the baryon and strange chemical potentials μ_{B} and μ_{s}, respectively [4–13]. Similar arguments are also be applied to the deconfined quarks and gluons in the QGP phase where the quark chemical potential $\mu_{\mathrm{q}}(=\mu_{\mathrm{B}}/3)$ and μ_{s} control the net quark (or baryon) and strangeness contents. In the framework of a statistical thermodynamical model such as the one described above we can construct a first order quark-hadron phase transition and obtain a phase diagram curve (i.e., critical $\mu_{\mathrm{B}}^{\mathrm{c}}$, T^{c}) in the μ_{B}-T plane for the co-existence of the HRG and QGP phases. This is achieved by imposing the Gibbs criteria of simultaneous thermal, chemical and mechanical equilibrium i.e., $T^{\mathrm{H}} = T^{\mathrm{Q}}$, $\mu_{\mathrm{B}}^{\mathrm{H}} = 3\mu_{\mathrm{q}}^{\mathrm{Q}}$ and $P^{\mathrm{H}} = P^{\mathrm{Q}}$ where the indices H and Q indicate the hadronic and the quark phases, respectively and P the pressure. The study of a first order phase transition between the QGP (with thermally and chemically equilibrated u, d and s quark flavours and gluons) and the HRG (where non-strange as well as strange hadrons have achieved a reasonable degree of thermal and chemical equilibrium) is of great importance. This is mainly due to the fact that according to the Gibbs criteria not only $\mu_{\rm B}$ but the $\mu_{\rm s}$ also should match across the phase boundary hence requiring that $\mu_{\rm s}^{\rm H} = \mu_{\rm s}^{\rm Q}$, in addition to the above mentioned conditions. This will thus establish a complete chemical equilibrium between the strange as well as non-strange sectors of the two phases. In the deconfined phase of quarks and gluons i.e., the QGP, the strange quark-antiquark pairs ($s\bar{s}$) are created/annihilated freely via the processes like $g\bar{g} \leftrightarrow s\bar{s}$, $q\bar{q} \leftrightarrow s\bar{s}$ [3–6] where $q(\bar{q})$ and $g(\bar{g})$ stand for light u, d quarks (antiquarks) and gluons (antigluons), respectively. In order to have zero net strangeness in the QGP phase one must set $\mu_{\rm s}^{\rm Q} = 0$ for all values of $\mu_{\rm q}^{\rm Q}$ and T. The deconfinement property of the quarks actually makes the $\mu_{\rm s}^{\rm Q}$ independent of $\mu_{\rm q}^{\rm Q}$ and T. However, the problem arises for the HRG phase when the criteria of strict strangeness conservation is applied. It occurs since the quarks are confined into various kind of hadronic resonances. One finds that the value of $\mu_{\rm s}^{\rm H}$ becomes a sensitive function of $\mu_{\rm B}^{\rm H}$ and T and we do not obtain $\mu_{\rm s}^{\rm H} = 0$ always for all possible values of $\mu_{\rm B}^{\rm H}$ and T except for $\mu_{\rm B}^{\rm H} = 0$, which gives zero net strangeness in the HRG phase [3–6]. However, quite interestingly it has been pointed out [13] that for certain critical values of $\mu_{\rm B}^{\rm H}$, T (where $\mu_{\rm B}^{\rm H} = 0$ is *not necessarily required*) it is possible to achieve the total zero strangeness in the HRG phase while the system still maintains $\mu_{\rm s}^{\rm H} = 0$. One can therefore obtain a strangeness conserving critical $\mu_{\rm B}^{\rm H}$, T phase diagram curve for the HRG phase by setting $\mu_{\rm s}^{\rm H} = 0$. Such a critical curve was shown to lie close to the first order phase diagram curve [13], with $\mu_{\rm s}^{\rm H} = \mu_{\rm s}^{\rm Q} = 0$. It should be noted here that under this condition the net strangeness content in the HRG phase may not be exactly zero for the critical $\mu_{\rm B}^{\rm c}$, $T^{\rm c}$ values obtained by the construction of a first order quark-hadron phase transition unless these critical values happen to exactly coincide with those obtained from the strangeness conservation criteria in the HRG phase with $\mu_{\rm s}^{\rm H} = 0$. Neverthless, the close proximity of the two critical curves leads to an optimistic conclusion that it may be possible for the HRG phase (consisting of non-strange hadrons as well as strange mesons and hyperons) to negotiate a first order phase transition to a QGP phase (with u, d and s quark flavours in thermochemical equilibrium) with strictly conserved strangeness. The previous study [13] was done in the context of the strange particle production in relativistic nucleus-nucleus collisions employing the HRG and QGP formalism. It was shown that the analysis of the heavy-ion CERN-SPS data gives the value of $\mu_{\rm s} \approx 0$ for the system, a condition which may not only occur in the QGP phase but in the HRG phase as well. In the analysis the hadrons were, however, regarded as point particles. This assumption is quite unrealistic and in fact leads to a very unphysical situation where one finds that according to the Gibbs criteria there can also exist another first order phase transition for larger values of $\mu_{\rm B}^{\rm c}$ and $T^{\rm c}$ where HRG phase becomes more stable than the QGP phase and the system reverts back to the HRG phase [14–19]. The effect becomes more prominent by the inclusion of more hadronic resonances in the HRG phase. This was shown to occur due to the ignorance of the effective finite-size of the hadronic resonances which mainly arises out of the well known property of the hadronic interactions at high density and temperature viz., the short range hard-core repulsion. This leads to the excluded volume effect and the equation of state (EOS) of the HRG phase is significantly modified, thus completely avoiding the possibility of the unphysical phase transition to HRG phase from the QGP phase in the region of large $\mu_{\rm B}$ and T, and the matter prefers to remain in the QGP phase. Here one may also recall an alternative hadronization mechanism suggested by Greiner et al. [20] for a baryon rich QGP, leading to the formation of metastable blobs of strange-quark matter, called strangelets. This mainly arises due to unequal hadronization rates for the s and $\bar{s}$ quarks in a baryon rich QGP. According to this mechanism, since the QGP contains light quarks (u, d) which are for more abundant than the light antiquarks $(\bar{u}, \bar{d})$ hence the emission of Kaons ($q\bar{s}$) from the QGP blob will occur at a much faster rate than those of antikaons ($\bar{q}s$). The possibility of separating strange quarks from antistrange quarks in the QGP $\leftrightarrow$ HRG transition will therefore cause a continuous enrichment of strange quarks in the QGP. Consequently the strange quarks in the QGP phase will acquire a chemical potential $\mu_{\rm s}^{\rm Q}$ different from 0. In the above picture it should also be realized that a rapid depletion of anti-strangeness from the QGP via $K(q\bar{s})$ emission will also cause a simultaneous rapid depletion of the baryon content of the QGP which will appear in the hadronic sector of the mixed phase. As a result $\mu_{\rm B}$ and $\mu_{\rm s}$ for the hadronic as well as QGP phase will continuously change with time during the evolution of the system via a mixed phase and consequently the chemical equilibrium conditions viz. $\mu_{\rm B}^{\rm H} = \mu_{\rm B}^{\rm Q}$ and $\mu_{\rm s}^{\rm H} = \mu_{\rm s}^{\rm Q}$ will not be satisfied during the transition. Hence it is to be emphasized that this would be totally inconsistent with the Gibbs equilibrium conditions for a first order phase transition [20]. Therefore the proposed hadronization picture will be far from a true first order quark-hadron phase transition process. Furthermore, Greiner et al. [20] in their model have though allowed for the strange chemical potential to evolve "continuously" with time during the phase transition, the baryon chemical potential is kept fixed, thereby resulting in an internal inconsistency in the light of the above discussion. In the above picture the effect of the hyperon production on the strangeness balance condition was ignored during the earlier part of the hadronization process. However, this does not seem to be physically justifiable since it is not clear why baryon rich QGP (i.e. with large baryon number) should not produce large number of strange as well as non-strange baryons continuously right from the onset of the hadronization process. In fact if one allows for a continuous hadronization of hyperons as well as Kaons, then it may result in a nearly equal hadronization rate for the s and $\bar{s}$ quarks and the QGP might still maintain $\mu_{\rm s}^{\rm Q} = 0$ and a given value of the quark chemical potential $\mu_{\rm q}^{\rm Q}$ during the transition, thus satisfying the Gibbs equilibrium conditions. As discussed earlier it has been found that for pointlike hadrons a strangeness conserving equilibrium first order quark-hadron phase transition can occur with $\mu_{\rm s}^{\rm H} = \mu_{\rm s}^{\rm Q} = 0$. The motivation of this paper therefore is that in view of all these facts it is of paramount importance to investigate that whether an equilibrium first order hadron-quark matter phase transition could still occur with strictly conserved strangeness by making use of a more realistic EOS for the HRG phase which will take into account the hard-core repulsive hadronic interactions leading to a finite-size effect. In Sect. 2 we discuss the modified EOS of the HRG and its different cases. On the QGP side we will employ an EOS which takes into account the perturbative quark gluon interactions of order g^2 and g^3 [19,21]. It has been shown earlier [19] that for the $\mu_{\rm B}$, T region where a first order quark-hadron phase transition may occur, the contribution of these interactions plays an important role in determining the location of the first order phase diagram curve in the $\mu_{\rm B}$-T plane. In Sect. 3 we present the EOS for the QGP phase. In Sect. 4 we will discuss the results and in Sect. 5 we will summarise and conclude.

2 The HRG phase

2.1 Pointlike hadrons with $\mu_{\rm s}^{\rm H} = 0$

It has been shown earlier [4–13] that if the HRG phase is regarded as a mixture of various non-interacting pointlike hadronic resonances then in the framework of the grand canonical ensemble theory one can derive the EOS of the system. This is done by writing the total partition function as a sum of the strange and non-strange sectors i.e.,

$$\ln Z = \ln Z^{\rm strange} + \ln Z^{\rm non\text{-}strange} \tag{1}$$

The abundance of strange (antistrange) hadrons can be obtained from the $\ln Z^{\rm strange}$ only. The chemical potentials of all hadrons are defined as [4–13]

$$\begin{aligned}\mu_i &= (q_i - \bar{q}_i)\mu_{\rm q} + (s_i - \bar{s}_i)\mu_{\rm s} \\ &= N_{\rm q}\mu_{\rm q} + N_{\rm s}\mu_{\rm s}\end{aligned}$$

where $N_{\rm q}$ and $N_{\rm s}$ are the number of valence light (u,d) and strange (s) quarks, respectively in the $i^{\rm th}$ type of hadronic species. The $\mu_{\rm q} = \mu_{\rm B}^{\rm H}/3$. This is sufficient to define the fugacities of all hadronic species e.g. it gives the Kaon fugacity $\lambda_{\rm K} = \lambda_{\rm q}\lambda_{\rm s}^{-1}$, antiKaon fugacity as $\lambda_{\rm q}^{-1}\lambda_{\rm s}$, non-strange baryon fugacities $\lambda_{\rm B} = \lambda_{\rm q}\lambda_{\rm q}\lambda_{\rm q}$, singly strange hyperon (Λ, Σ) fugacities as $\lambda_{\Lambda,\Sigma} = \lambda_{\rm q}\lambda_{\rm q}\lambda_{\rm s}$ etc. The number density of a pointlike hadronic species in the system can be obtained (by using Boltzman approximation for simplicity) as [4–6,12]

$$n_i = \frac{g_i}{2\pi^2} T^3 \lambda_i W(m_i/T) \tag{2}$$

where m_i and g_i are the mass and the spin-isopin degeneracy factor of the $i^{\rm th}$ type of hadronic species. The T is the thermal temperature of the system and $W(m_i/T) = (m_i/T)^2 K_2(m_i/T)$ with K_2 as the modified Bessel function. If we include in our system the singly strange mesons, hyperons and the doubly strange Ξ resonances we can write the strangeness balance condition for the HRG phase as

$$\begin{aligned}&\sum_K g_K W_K \left(\lambda_{\rm q}^{-1}\lambda_{\rm s} - \lambda_{\rm q}\lambda_{\rm s}^{-1}\right) \\ &\quad + \sum_Y g_Y W_Y \left(\lambda_{\rm q}^2\lambda_{\rm s} - \lambda_{\rm q}^{-2}\lambda_{\rm s}^{-1}\right) \\ &\quad + \sum_\Xi g_\Xi W_\Xi \left(\lambda_{\rm q}\lambda_{\rm s}^2 - \lambda_{\rm q}^{-1}\lambda_{\rm s}^{-2}\right) = 0\end{aligned} \tag{3}$$

Here K, Y and Ξ stand for Kaons, singly strange hyperons and cascade resonances. One can immediately check that for the baryon free matter with $\mu_{\rm B} = 0$ (i.e., $\lambda_{\rm q} = 1$) we obtain $\lambda_{\rm s} = 1$ or $\mu_{\rm s} = 0$. However, it is also possible to find, as discussed earlier, a set of solutions for $\mu_{\rm q}$ in a certain range of T for which $\mu_{\rm s} = 0$ always [13]. Setting $\lambda_{\rm s} = 1$ and solving for $\mu_{\rm q}$ we get

$$\mu_{\rm q} = T\cosh^{-1}\left\{\frac{\sum g_K W_K - \sum g_\Xi W_\Xi}{2\sum g_Y W_Y}\right\} \tag{4}$$

Using (4) we can obtain a critical $\mu_{\rm q}$, T curve for $\mu_{\rm s} = 0$ with strictly conserved strangeness in the HRG phase consisting of pointlike hadronic resonances.

2.2 Finite size hadrons with $\mu_{\rm s}^{\rm H} = 0$

As discussed in Sect. 1 we now incorporate the effect of the short-range hard-core repulsive interaction among hadrons. To achieve this in a phenomonological model the hadrons are regarded as hard incompressible but deformable bags which cannot penetrate each other thus giving rise to an excluded volume effect [14–19]. This leads to an EOS where thermodynamic quantities are modified by a multiplicative factor. For example, the number density of $i^{\rm th}$ type of "finite-size" hadronic species becomes [14–19]

$$n_i = n_i^o\left(1 + \sum n_j^o v_j\right)^{-1} \tag{5}$$

where $n_j^o(n_i^o)$ is the number density of the $j^{\rm th}$ ($i^{\rm th}$) "pointlike" hadron given by (2) and v_j the hard-core volume of the corresponding finite-size hadron. The summation in (5) runs over all the hadronic species in the system with hard-core repulsion existing among them. The above correction factor is also applied to the pressure function e.g. the partial pressure due to the $i^{\rm th}$ type of hadronic species is given as

$$P_i = P_i^o\left(1 + \sum n_j^o v_j\right)^{-1} \tag{6}$$

If the summation in the correction factor in (5) is assumed to include all the mesonic as well as baryonic (antibaryonic) degrees of freedom including the strange (antistrange) ones then the correction factor in the number density will become same for all types of hadronic species and hence the condition of strangeness balance in the HRG phase with $\mu_{\rm s}^{\rm H} = 0$ will again yield the same result for $\mu_{\rm q}$ as obtained for the case of pointlike hadrons given by (4). However, this assumption is not very convincing for certain reasons, because it is believed that the hard-core repulsion exists only between a pair of baryons (or antibaryons) while it is of attractive nature for a baryon-antibaryon, meson-baryon and meson-meson pairs [18,19]. Thus one should apply separate corrections to the baryon and antibaryon sectors while the mesons are assumed to be free from such hard-core repulsion and thus their contribution either to the strangeness content or pressure of the system should not apparently require any modification. It may be noted here that this modified EOS is actually thermodynamically inconsistent. However, it does not lead to any appreciable error in the calculation of the number densities [7,22] and besides its application is quite simple. This kind of EOS will thus serve our purpose which is to illustrate the effect of the inclusion of the finite-size of hadrons on the first order quark-hadron phase transition with $\mu_s^H = 0$ and conserved strangeness. In the above picture however one may further require to distinguish between two different physical situations, case 1: when the thermal mesons are allowed to move over the entire region including those occupied by the baryons (antibaryons) and case 2: when they can move freely only in the available volume i.e., in the region not occupied by baryons (antibaryons). In the case 1 the contribution of Kaons to the net strangeness content is not affected by the finite-size of baryons (antibaryons) while in case 2 it is modified (reduced) by the fraction of the occupied volume or in other words their contribution is proportional to the fraction of the available volume, although they do not exhibit any hard-core repulsion. Recently it was found that in order to explain the CERN data in the framework of a recently proposed thermodynamically consistent EOS for the HRG phase it is necessary to assume that the thermal mesons can exist only in the region not occupied by the baryons (antibaryons) [9–11,23]. In the present analysis also, as we shall discuss later, only the second scenario (i.e. the case 2) leads to a favourable physical situation. It is found that there does not exist at all any physical phase diagram for the case 1 if we set $\mu_s^H = 0$ and demand strangeness conservation while for case 2 it is possible to find a physical solution for μ_B^H and T, which we can obtain numerically only. It should be stressed here again that in the case 2 the non-availability of the regions occupied by the baryons (antibaryons) to the "thermal" mesons in the fireball is "not" due to any hard-core repulsion between mesons and baryons (antibaryons). It is the out come of the possibility that if the mesons happen to penetrate into these regions then they can be easily absorbed leading to various possible reaction processes, and therefore these mesons will not remain in a "pure thermal state" or simply speaking in a state where their thermal momentum spectra can be described by the Bose-Einstein distribution functions. On the other hand if they come close to these regions but do not penetrate then due to the long range part of the strong force they will be only scattered and hence will continue to remain in a thermal state.

3 The QGP phase

In the deconfined phase of quarks and gluons the strangeness can be conserved by setting $\mu_s^Q = 0$. This is a sufficient condition. Under this condition the number density of strange and antistrange quarks will become equal. It is also necessary to incorporate the effect of interactions in the deconfined quark gluon phase, since the lattice QCD [1,13,24] and some phenomenological results [25] employing mean field theory chiral calculations in dense nuclear matter, indicate that unless the temperature in the QGP phase is very high $\sim 250\,\text{MeV} \approx 2\,T_c$ (where T_c is the critical temperature for deconfinement at zero baryon chemical potential) the quarks and gluons will continue to interact at least perturbatively. In a recent work of Le Bellac and Braaten and Pisarski [26,27] some of these aspects have been discussed. Boyanovski et al. [28] have also considered an approach to the dynamics of relaxation and kinetics of thermalization in a scalar field theory incorporating the contributions of hard thermal loops. They employ the non-equilibrium quantum field theory to study the relaxation, in the quark and the hadron phases. However, in order to construct a proper first order quark-hadron phase transition we will require to write the QGP pressure function by taking into account the perturbative interactions. We assume that both the HRG and QGP phases have reached a reasonable degree of thermal and chemical equilibrium. For determining the total partition function (or pressure) due to various quark flavours and gluons in the QGP phase, it is convenient to employ an EOS for the QGP phase suggested by Kapusta [21]. This incorporates the g^2 and g^3 contribution in the strong interaction coupling, which was obtained by the application of the finite temperature field theory (FTFT). This provides us a more realistic EOS for the QGP phase [19] since at the phase boundary the perturbative interactions among the quarks and gluons contribute significantly to the QGP pressure. Although, the lattice QCD numerical results indicate that at and above the phase boundary, for deconfinement phase transition, the quarks and gluons may still interact somewhat non-perturbatively. Nevertheless we for the sake of simple phenomenological approach assume (and as will be seen later) that the critical phase boundary obtained by considering perturbative interactions in the EOS of the QGP upto order g^3 and the above discussed EOS for the HRG will lead to a situation which is not very far from what is indicated by some early lattice QCD numerical simulation results [1,13,24,26]. We write the total partition function as [21]

$$\begin{aligned}\frac{T\ln Z}{V} = \frac{2N_c}{3}\sum_f \int \frac{d^3p p^2}{(2\pi)^3}\frac{n_p}{E_p} \\ -\frac{1}{3}\pi\alpha_s N_g T^2 \sum_f \int \frac{d^3p}{(2\pi)^3}\frac{n_p}{E_p} \\ -\pi\alpha_s N_g \sum_f \int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{1}{E_p \cdot E_q} \\ \left[\frac{2m^2}{(E_p - E_q)^2 - \omega^2} + 1\right] n_p \cdot n_q\end{aligned}$$

$$+\frac{\alpha_s^{3/2} N_g T}{3\pi^2\sqrt{2\pi}}\left[\frac{2}{3}\pi^2 N_c T^2 + \sum_f \int dp(p^2+E_p^2)\cdot\frac{n_p}{E_p}\right]^{3/2} + \frac{N_g \pi^2 T^4}{45} - \frac{1}{36}\alpha_s \pi N_c N_g T^4 - B \quad (7)$$

In the above expression

$$n_p = n_p^+ + n_p^- \; ; \; n_q = n_q^+ + n_q^- \; ; \; \omega = |\mathbf{p}-\mathbf{q}|$$

where n_p^+ (n_q^+) and n_p^- (n_q^-) represent fermion and antifermion distribution functions, respectively. The N_c and N_g are the number of quark and gluon colours, respectively. The summation over f is for the various quark flavours which we take as u, d and s. The quantity $\alpha_s = g^2/4\pi$ represents the running coupling constant in FTFT [21] and is given as

$$\alpha_s = \frac{4\pi}{(11-\frac{2}{3}N_f)\ln(M^2/\Lambda^2)} \quad (8)$$

where Λ is the QCD scale fixing parameter and N_f the total number of quark flavours. We use $\Lambda = 160\,\text{MeV}$ [19]. The quantity M for a thermalized QGP phase is given as [19,21]

$$M^2 = (4/3)\frac{[N_c\sum_f\int dp\, p^4 n_p + N_g\int dp\, p^4 N_p]}{[N_c\sum_f\int dp\, p^2 n_p + N_g\int dp\, p^2 N_p]} \quad (9)$$

Here N_p is the gluon distribution function. Hence in the present analysis the running coupling constant α_s is function of μ_q and T, which was not the case in the earlier analysis [13] where it was chosen to be a constant, for determining the quark-hadron phase boundary. In the following we discuss the results of calculations. In (7) B is the bag constant. Here it is assumed that the entire QGP matter exists inside a big bag. This according to the Bag Model is required, in order to take into account the confinement property of the quarks and gluons. Hence B essentially accounts for the non-perturbative feature of the QCD interaction.

4 Results and discussion

As described earlier in Sect. 2 we will clearly distinguish between two possible physical situations for the application of the EOS of the HRG phase consisting of finite-size hadrons viz., case 1, when thermal mesons (including Kaons) are allowed to move over the entire region of space and their abundance is not affected by the presence of finite-size baryons and antibaryons which occupy a certain fraction of the total volume of the system and case 2, where they can exist freely (i.e. in thermal state) only in the available volume and hence their abundance is affected (reduced) by the fraction of the volume occupied by the baryons and antibaryons. The case 2 may lead us

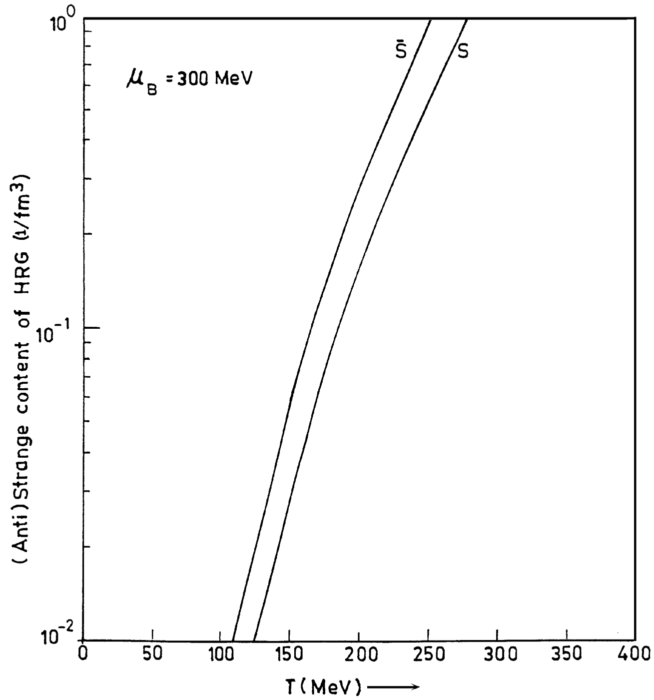

Fig. 1. Strange and antistrange contents of the HRG indicated by S and $\bar{S}$, respectively, with hadrons up to 1410 MeV mass. Here all mesons including Kaons are assumed to be free of hard-core repulsive interactions which is applied to (anti)baryons. Furthermore the Kaons are allowed to penetrate through the entire volume including that occupied by the (anti)baryons

to a more realistic situation as we have already discussed in Sect. 2 and as we shall find later in the present analysis. Before proceeding further it is also necesary to define the set of our input particles $\{i : g_i, m_i)$ which reflect the hadron mass spectrum considered in the analysis and thus describe the actual composition of the HRG system. The hadrons with masses up to 1410 MeV are incorporated in the system. This contains singly and doubly strange particles besides other nonstrange hadrons. The omission of the further heavier resonance states is done mainly due to the fact that the process of their chemical equilibration will be extremely slow and thus long enough to even partially fill the phase space during the lifetime of the hot fireball formed in the present day ultra-relativistic nucleus-nucleus collisions [4,9,10]. Hence their abundance can be safely ignored compared to the lighter hadrons formed at the temperatures $\sim 150\,\text{MeV}$ or so. The mass suppression effect can also be approximately estimated by the Boltzmann factor $e^{-m/T}$, where T is the temperature of the system. For cascade this factor is quite small $\sim 10^{-4}$ while for omega it is an order of magnitude further smaller $\sim 10^{-5}$. For the case 1 it is found that if we set $\mu_s^H = 0$ in the HRG phase then it is not at all possible to find any real physical solutions for μ_B^H and T which will strictly conserve the strangeness. This is well illustrated in Fig. 1 where the two curves corresponding to strange (S) and antistrange ($\bar{S}$) content of the system are separately plotted with T for a fixed value of $\mu_B^H = 300\,\text{MeV}$. We see that the two curves do not intersect, thereby implying that $S - \bar{S} \neq 0$ anywhere. We therefore find that case 1 may not be of our interest here. It, however, does imply one thing that if the meson's abundance is not affected by the presence of baryons and antibaryons then the system cannot have $\mu_s^H = 0$ except for a special case when $\mu_B^H = 0$ (i.e., baryon symmetric matter) and hence no other critical set for μ_B^H and T exist except for $(\mu_B^H = 0, T)$, as pointed in Sect. 1. In physical terms this essentially happens because while on one hand the strange hyperons (antihyperons), have their abundances affected by the finite size of all the baryons (antibaryons), the Kaon's abundance is not modified and the distribution of the strangeness contents in various mesonic and baryonic degrees of freedom of the system becomes such that it is impossible to achieve the condition of zero net strange with $\mu_s^H = 0$ and $\mu_B^H \neq 0$, which corresponds to baryon asymmetric HRG phase. Hence the values of $\mu_s^H \neq 0$ are required which are not of our interest here since we are dealing with the possibility of a first-order quark-hadron phase transition where $\mu_s^H = \mu_s^Q = 0$ is required along with strictly conserved strangeness. The case 2, however, indeed turns out to be a favourable one. It is found that it is possible to obtain physically acceptable solutions of μ_B^H and T for such a situation (viz. $\mu_s^H = 0$). This actually arises out of a different physical situation where Kaon's contribution is affected by the presence of the other baryonic resonances. In Fig. 2 the dashed-crossed curve shows the critical curve for such a case. Moreover, though unphysical but in the same figure, also plotted are the results of calculations for pointlike hadrons (4) for the sake of comparison with the case of finite-size baryons. Here we consider two situations. The solid curve corresponds to a simple case of a few ground state pointlike resonances in the strange sector of the HRG phase viz, Λ, Σ, Ξ and K while the dashed curve shows the results when pointlike hadrons up to 1410 MeV mass are incorporated in the system. It is very interesting to note that the curves for the above two pointlike cases are almost completely overlapping in the entire μ_B and T region. It therefore probably suggests us that the behaviour of the system with regard to the critical values of μ_B^H and T for $\mu_s^H = 0$ and zero net strangeness is not significantly affected by the number of hadronic resonances in the HRG phase. Furthermore, equally interesting is the observation that the consideration of the finite-size (anti) baryon resonances (case 2) also does not alter the critical behaviour of the system significantly, with regard to the strangeness conservation constraints in the HRG phase, and the dashed-crossed curve for the case 2 is seen to be almost completely overlapped by the two pointlike hadron cases discussed above. This, however, does not mean that the consideration of finite-size effect is not important in the context of the present analysis since it greatly affects the phase diagram curve obtained by constructing a first order quark-hadron phase transition. Here it is also very important to note that in this analysis the hard-core volumes of baryons (antibaryons) and the bag constant B used in the QGP phase EOS (7) are closely related from the bag model considerations [9–11,19,23], where the en-

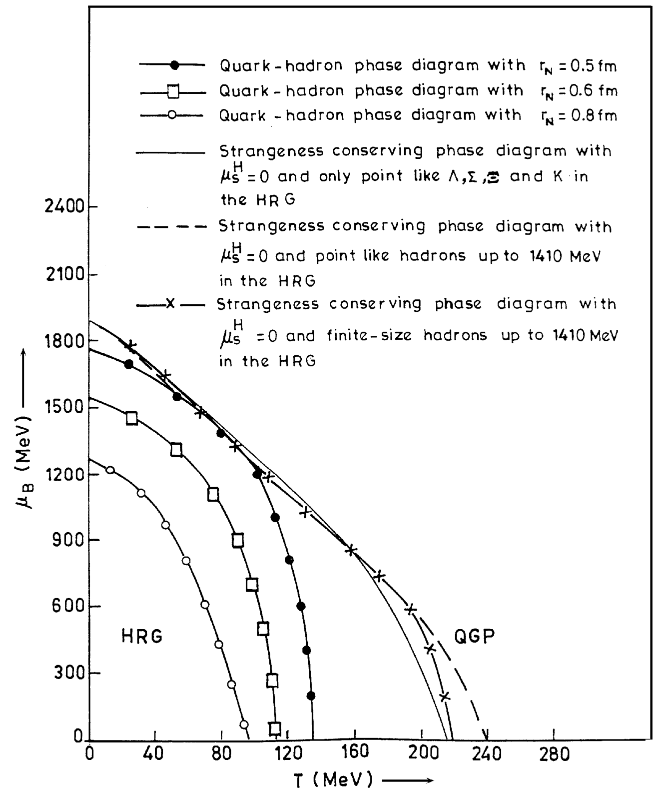

Fig. 2. Critical (μ_B^c, T^c) curves for various cases. The solid and dashed curves are for the case of strangeness conservation in the HRG phase with $\mu_s^H = 0$ and pointlike ground state Λ, Σ, Ξ and K only (solid) and hadrons up to 1410 MeV mass (dashed). The dashed-crossed curve is again for the case of zero net strangeness in the HRG phase with $\mu_s^H = 0$ but with finite-size (anti) baryons ($r_N = 0.8$ fm) up to 1410 MeV mass while the thermal mesons are allowed to exist only in the volume not occupied by the (anti) baryons (i.e., case 2). The dashed-open dircle, dashed-box and dashed-filled circle curves represent the first order quark-hadron phase transition boundary with $\mu_s^H = \mu_s^Q = 0$ for the three values of $r_N = 0.8$, 0.6 and 0.5 fm, respectively and with hadrons up to 1410 MeV

ergy density inside a hadron is given by $4B$. By selecting a suitable value, say, for the nucleon hard-core radius (r_N), we can fix the bag constant by using $4B = M_N/v_N$, where M_N is the nucleon mass which gives the total energy of the hadron bag and the nucleon hard-core (bag) volume $v_N = (4/3)\pi r_N^3$. This in turn will also simultaneously fix the hard-core volumes ($v_j s$ in (5)) of the other heavier baryonic (antibaryonic) resonances (mass m_j) by using $v_j = m_j/4B$. In this way we have thus reduced a large number of arbitrary parameters to just one (i.e. r_N) in the theory. In Fig. 2 three first order quark-hadron phase diagram curves (for $\mu_s^H = 0$) are shown for the three choices of r_N viz., 0.8 fm (represented by dashed-open circle curve), 0.6 fm (represented by dashed-box curve) and 0.5 fm (represented by dashed-filled circle curve). We regard the larger value of 0.8 fm as the possible upper bound and the smaller value of 0.5 fm as the lowest bound for r_N. The three curves are seen to be well separated. The corresponding values of B for these cases come out to be 110, 260 and 450 MeV/fm^3. Thus we find that smaller the value of r_N larger is the bag constant. This has two fold effect on the critical μ_B and T values for the first order phase diagram curve. Firstly, that with smaller r_N the pressure in the HRG phase tends to become larger at any given value of μ_B^H and T (cf. (6)) and secondly the corresponding larger value of B will tend to stronger confinement effects in the QGP phase thereby decreasing the net pressure of the system for the same given values of μ_B and T. As a result the first order quark-hadron phase trantition for smaller values of r_N requires larger critical values of μ_B and T to occur. This is well exhibited in Fig. 2 by the shifting of the phase diagram curves towards larger values of μ_B and T as r_N is made smaller. Here we also notice that by and large all the three curves are quite far away from the region where the cirtical curves for the HRG phase with $\mu_s^H = 0$ and zero net strangeness lie, except for a narrow range of μ_B and T where the quark-hadron phase diagram curve for $r_N = 0.5$ fm is seen touching this region. Hence (except this) at these phase boundaries the HRG system will not maintain the condition of strangeness neutrality. Here it is worthwhile to note that if we accept 0.5 fm as the lower acceptable bound for r_N it results in a very large value of B ($= 450$ MeV/fm^3) and it will yield energy density inside a hadron $= 4B = 1.8$ GeV/fm^3. This is an extremely large value. We therefore tend to accept this falue of r_N with caution. The other choices of r_N seem reasonable in particular 0.8 fm [9–11] The dashed-crossed critical curve for the HRG phase with $\mu_s^H = 0$ and zero net strangeness is for $r_N = 0.8$ fm. As is obvious from the comparison (with the pointlike cases i.e., $r_N = 0$ it will almost completely overlap with the critical curves (not shown in the figure) obtained by setting $r_N = 0.6$ and 0.5 fm, as done for obtaining the quark-hadron phase diagram curves.

Though earlier some other methods were used to determine the baryon radii viz. using the Regge trajectories [29,30], but given the great simplicity of doing this we have chosen to invoke the bag model which has also successfully been used to describe the hadron mass spectrum. It is also necessary to invoke the existence of a bag in order to construct a reasonable quark-hadron phase transition [4,12,19,26]. Moreover, in the Regge approach the possibility of the elementary particles (i.e. quarks and gluon) was eliminated and all the baryons (and mesons as well) with the same spin have almost the same radius, even if their masses are greatly different and/or they carry the strange quark e.g. for spin 1/2 baryons ≈ 0.7 fm [29,30]. Our main aim here is to determine that what is the extent of the role played by the geometrical effects viz. the hard-core repulsions leading to finite-size and the excluded volume effects. The way of the determination of the exact radii for various cases of baryon-baryon pair hard-core interaction is therefore not of great concern here. Further, in order to test the sensitivity of the results and conclusions we have chosen several values of the nucleon (and consequently those of the other baryons) radii and found that the conclusions are quite valid over the entire range of these values for $r_N \geq 0.5$ fm.

It is to be noted that for a baryon free matter ($\mu_B = 0$) the value of T_c obtained for the first order quark-hadron phase transition (with r_N having a value around 0.6 fm) is ≈ 120 MeV which is in a reasonably good agreement with the earlier lattice QCD results [1,13,24,26]. It therefore seems to provide some support to the kind of EOS's employed in the present study, as hinted in Sect. 3. However, the columbia group [31] QCD numerical simulation results for staggered quarks at $N_t = 4$ with 16^3 lattices indicate that for two ($N_F = 2$) massless flavours (u, d) the transition seems to be of second order and above certain light quark mass no transition occurs. For three ($N_F = 3$) degenerate flavours (u, d, s) the transition is of first order while above certain light and strange quark masses ($m_u = m_d = 12$ MeV, $m_s = 50$ MeV) again no transition is observed. Thus for $N_F = 2 + 1$ with nearly massless u, d quarks there is a strange quark mass above which no transition occurs while below this a first order phase transition is seen. Some earlier results of Ukawa [32] and Fukugita [33] have also shown that no transition occurs for moderately massive quarks. Moreover, in contrast to this Iwasaki et al. [34] using Wilson quarks have recently tried to show that for $N_F = 2+1$ a first order QCD phase transition occurs even for $m_s = 400$ MeV on a $12^3 \times 4$ lattice.

However, the cited work of Iwasaki et al. has got certain flaws and can not be taken seriously. Iwasaki et al. took the divergence of their numerical algorithm above certain temperature as signaling a physical phase transition. This however is a bad criterion, because that would mean they were not able to have physically acceptable results for temperature above certain point. This divergence therefore can not be taken as an indication of a physical phase transition. On the other hand the earlier works by Columbia group have all the calculation converging at every temperature they have investigated. In summary the reliable lattice numerical results indicate an absence of a first-order phase transition in the real world when the s quark is moderately heavy (> 50 MeV).

In view of the above a descrepancy arises since there are many phenomenological model calculations (employing various types of EOS's) which do indicate the existence of a first order quark-hadron phase transition for $m_s = 150$ MeV. This descrepancy seems to arise because of different approaches. For example in the phenomenological model calculations we require to write down the explicit EOS's for the HRG as well as the QGP phase separately. On the other hand the lattice simulation of the QCD does not provide a tractable EOS for the HRG phase. Hence the descriptions of the hadronic phase are different in the phenomenological and the lattice QCD approaches. Further, the phenomenological approach (unlike the lattice QCD approach) invokes the Gibbs criteria of the co-existence of the two phases for the construction of a phase transition while simultaneously conserving the net strangeness. These inherent differences in the two approaches seem to give rise to very different expectations with regard to the existence of a phase transition when the s quark mass is > 50 MeV. Therefore the results obtained in the present calculation are to be viewed in the background of the phenomenological approach.

The above results thus indicate that under the physically acceptable conditions the first order quark-hadron phase diagram curve, with $\mu_s^H = \mu_s^Q = 0$, is not expected to overlap with the critical curve for the HRG phase with $\mu_s^H = 0$ and zero net strangeness. It therefore does not seem physically possible for a thermo-chemically equilibrated hadronic resonance matter to negotiate a first order equilibrium phase transition to another thermochemically equilibrated QGP phase with strictly conserved strangeness.

The present study clearly indicates that the strangeness conservation constraints are crucial and may actually hinder the possibility of an equilibrium quark-hadron phase transition as expected in the ultra-relativistic heavy-ion collision or during the early universe, if one considers a more realistic set of EOS for the HRG and the QGP phases. However, in the presence of some non-equilibrium effects e.g. partial thermochemical equilibrium [7–12] of various hadron resonances species, early loss or leakage/ evaporation of strangeness (antistrangeness) content of the system, before the occurrence of the actual first order equilibrium phase transition, and more importantly due to a superheating in the HRG and supercooling effect in the QGP phase, there may still occur a QGP $\leftrightarrow$ HRG phase transition. In other words under such physical situations the matter may negotiate a nearly (or weak) first order phase transition with conserved strangeness. Here the early loss of strangeness (antistrangeness) content prior to the formation of a thermally and chemically equilibrated QGP fireball is not to be misunderstood with the scenario proposed by Greiner et al. [20]. Here we only permit a nonzero value of the net strangeness content i.e. $(S - \bar{S} \neq 0)$ giving rise to a non-zero value of μ_s^Q which "remains constant" during the later part of the evolution of the QGP fireball and also while the system is negotiating an equilibrium first order phase transition. Under these special conditions the system may cross the region of large separation, seen in Fig. 2, between the phase diagram curves and the strangeness conserving critical curves.

5 Summary and conclusion

In summary we have investigated the possibility of a strict first order deconfinement phase transition from an equilibrated hadronic resonance gas to another phase of equilibrated, perturbatively interacting plasma of quarks and gluons with strictly conserved strangeness. We have considered a more realistic physical situation where the phenomenological EOS of the hadronic resonance matter incorporates an important feature of hadronic interactions at high density and/or temperature viz., hard-core repulsive interaction leading to a finite-size effect. We further assume the hard-core repulsion to exist between a pair of baryons or antibaryons while mesons are free from such repulsive interaction. In the QGP phase we have considered perturbative interactions upto the order g^2 and g^3 in the strong coupling. We construct a first order quark-hadron phase transition by employing the Gibbs criteria and determine the critical values of μ_B and T. Simultaneously we also determine a set of critical values of μ_B^H and T for the hadronic phase by demanding the conservation of strangeness and setting $\mu_s^H = 0$ (as required by the Gibb's criteria). It is found that real physical solutions can exist only under a plausible assumption that the region of space occupied by the baryons and antibaryons is not available to the "thermal mesons". We find that the two phase diagram curves in the μ_B-T plane are far apart for physically acceptable values of the baryon hard-core radii and the bag constant B which are related from the bag model considerations (due to which the number of arbitrary parameters in the model is also reduced to *only one* i.e., the nucleon hard-core radius r_N). The above results contradict the previous results for ideal pointlike hardons where the two phase diagram curves were found to lie close to each other in the μ_B-T plane, hence indicating the possibility of QGP formation in the CERN experiment. Though providing interesting results we, however, regard the case of pointlike hadrons as unphysical since in this case there exists no reasonable phase transition to the QGP phase at high temperatures or baryon density (or chemical potential) due to the possible excitation of a vast number of hadronic resonances. Our aim has been to study that upto what extent the strangeness conservation constraints and the finite baryon (antibaryon) sizes may play an important role in the first order equilibrium quark-hadron phase transition. The present more realistic and phenomenological approach leads to the conclusion that it is quite less likely for a strongly interacting matter consisting of strange as well as non-strange hadronic resonances to negotiate a first order equilibrium phase transition to another phase of deconfined quarks and gluons with conserved strangeness, except for some special physical situations where nonequilibrium effects may become important.

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